On the realizability of magnetohydroelectric interaction

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A variant of the initially proposed magnetohydroelectric theory that may bring the theory within the realm of experimental verifiability and practical applicability is suggested.

In recent publications[1,2] we have outlined the theoretical basis of hydromagnetic interaction in a fluid dielectric in the presence of a steady magnetic field $\mathbf{B}_0$. This interaction was briefly termed magnetohydroelectric interaction. It was found that a theoretical development somewhat similar to the subject of magnetohydrodynamics is possible in this case. However, the values that the various physical parameters would have to have in order to make the magnetohydroelectric interaction observable appeared to be unrealistic in view of present-day technology. It was found, for instance, that the interaction becomes dominant (i.e., the electromagnetic energy and the kinetic energy of the fluid become comparable) when

$$Y = \chi\epsilon_0 B_0^2/\rho \gg 1 , \tag{1}$$

where χ and ρ are the dielectric polarizability and the density of the fluid, and ϵ_0 is the permittivity of free space. It immediately becomes obvious that the field $\mathbf{B}_0$ has to be unrealistically large in order to realize this condition. If this condition is not satisfied, only a small fraction of the electromagnetic energy couples into the fluid motion. Thus, the practical applicability of the proposed theory becomes questionable. We show, however, that with a slight modification of the basic assumptions in the theory, it is possible to couple arbitrary amounts of electromagnetic energy into the fluid motion even under the condition $Y \ll 1$.

The magnetohydroelectric volume force on a fluid is

$$\mathbf{F} = \mathbf{J}\times\mathbf{B} + \rho_c \mathbf{E} , \tag{2}$$

where $\mathbf{J}$ is the electric current measured in the laboratory frame, ρ_c is the electric space charge density in the fluid, $\mathbf{E}$ is the electric field in the laboratory frame, and $\mathbf{B}$ is the ambient magnetic field. If the fluid velocity $\mathbf{u}$ is much smaller than the velocity of light, then the current $\mathbf{J}$ may be obtained from the following relations[3]:

$$\mathbf{J} = \mathbf{J}' + \rho_c \mathbf{u} , \tag{3}$$

$$\mathbf{J}' = \chi\epsilon_0 \frac{\partial \mathbf{E}'}{\partial t'} , \tag{4}$$

$$\mathbf{E}' = \mathbf{E} + \mathbf{u}\times\mathbf{B} , \tag{5}$$

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + (\mathbf{u}\cdot\nabla) , \tag{6}$$

where the primed quantities are those measured in the frame of the moving fluid. From these relations we obtain

$$\mathbf{J} = \chi\epsilon_0\left(\frac{\partial \mathbf{E}}{\partial t} + \frac{\partial}{\partial t}(\mathbf{u}\times\mathbf{B}) + (\mathbf{u}\cdot\nabla)(\mathbf{E}+\mathbf{u}\times\mathbf{B})\right) + \rho_c \mathbf{u} . \tag{7}$$

Simple estimates now show that the force $\mathbf{F}$ becomes appreciable only at high frequency electric fields, but at these high frequencies the time scale is too short for the bulk fluid to move through macroscopic distances. Since the time average force on the fluid vanishes, the bulk fluid remains stationary. The only effect is a mechanical "jitter" of the molecules at a microscopic level at a frequency equal to the frequency ω of the electric field. Unless $Y \gg 1$, the fraction of the electromagnetic energy coupled into this jitter is very small.

Now consider a simplified experimental configuration as follows: Suppose the fluid is placed between the two conductors of a cylindrical capacitor (consisting of a hollow cylinder and a coaxial conducting wire or rod). An oscillating electric field $\mathbf{E}$ is applied between the two conductors, and an oscillating axial magnetic field $\mathbf{B}$ is applied by means of a solenoid wrapped around the cylinder. The resulting force $\mathbf{F}$ and the velocity $\mathbf{u}$ would then be in the toroidal direction (i.e., the ϕ direction of cylindrical coordinates). Note that with the geometry of the experiment the $(\mathbf{u}\cdot\nabla)$ term vanishes, and the force component arising from ρ_c is radially directed so that it merely creates a stress in the fluid. The electric field arising from the time varying magnetic field is toroidal, and does not affect our discussion. Suppose next that the fields $\mathbf{E}$ and $\mathbf{B}$ are synchronized such that

$$E = E_0 \sin\omega t , \tag{8}$$

$$B = B_0 \cos\omega t . \tag{9}$$

The net motive force on the fluid in the ϕ direction is then

$$F = \chi\epsilon_0(E_0 B_0 \omega \cos^2\omega t + B_0^2 \dot{u} \cos^2\omega t - \tfrac{1}{2} B_0^2 u\omega \sin 2\omega t) . \tag{10}$$

Setting $F = \rho\dot{u}$, we have

$$\dot{u} = \frac{(YE_0/B_0)\omega\cos^2\omega t - (Y/2)u\omega\sin 2\omega t}{1 - Y\cos^2\omega t} . \tag{11}$$

The ratio of the amptitude of the first term to that of the second in the numerator of the right-hand side is $2E_0/uB_0$. Clearly then, as long as u remains substantially smaller than $2E_0/B_0$, $\dot{u}$ is positive. This means that the fluid experiences a unidirectional volume force, the time average value of which is (with $Y \ll 1$)

$$\langle F\rangle = (\rho\omega E_0/2B_0)Y . \tag{12}$$

By a proper choice of parameters, this force can be made large enough to produce observable motion of the bulk fluid. Consider, for example, the case of chemically pure water ($\chi \approx 80$, with the imaginary part of χ being negligible at MHz frequencies). Let $\omega = 2\pi\times 10^6$ (a frequency of 1 MHz), $B_0 = 1$ Wb m^{-2} ($=10^4$ G), and $E_0 = 10^5$ Vm^{-1}. These lead to $\langle F\rangle \approx 2.2\times 10^2$ newton m^{-3}, or

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an acceleration that is about 2% of the acceleration due to gravity. This should be sufficient to set the fluid in motion even against viscous forces which we have not considered in this simple analysis. Note that our assumption of the neglect of the $\sin 2\omega t$ term in Eq. (11) remains valid here as long as $u \ll 2\times 10^5$ m sec^{-1}. If, and when, u attains this value, this term will come into play and the nature of the interaction will become complicated. Note also that the rotational bulk motion of the fluid in the cylinder will be accompanied by molecular jitters at the frequencies ω and 2ω. While the fraction of the electromagnetic energy coupled into the jitter is much smaller than unity, the ratio of the energy density of bulk motion to that in the electric field could, in principle, be arbitrarily large. Note lastly that oscillatory motion of the bulk fluid at some frequency $\omega_1 (\ll \omega)$ can be produced by simply reversing the polarity of the fields described in Eqs. (8) and (9) at this frequency synchronously.

Here, we have essentially outlined a simple thought experiment. Performing the actual experiment would entail a number of additional considerations. From our initial consideration, the suggested effect appears to be demonstrable. If so, one could imagine a number of practical applications of the effect of alternating field synchronous magnetohydroelectrics. The essence of such an application would consist in the capability of accelerating dielectric fluids to high velocities dependent on their dielectric constants, while the molecular dipoles undergo oscillations at the frequency of the applied electric field and its harmonic.

[1] B. R. De, Phys. Fluids 22, 189 (1979).
[2] B. R. De, Astrophys. Space Sci. 62, 255 (1979).
[3] J. A. Stratton, *Electromagnetic Theory* (McGraw-Hill, New York, 1941), p. 78.

Finite amplitude solitary Alfvén waves

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Solitary Alfvén waves with small but finite β have been studied by Yu and Shukla [Phys. Fluids **21**, 1457 (1978)]. A more rigorous study is made and conditions for uniqueness are established.

For Alfvén waves propagating in a plasma, Yu and Shukla[1] have obtained the following equation:

$$\frac{1}{2}\left(\frac{\partial\eta}{\partial\eta}\right)^2 + \pi(n) = 0\,, \tag{1}$$

where η is of the form

$$\eta = K_x x + K_y y - Vt \tag{2}$$

such that K_x, K_y, and V are constants, and $n = n(\eta) = n_i = n_e$, where n_i is the ion density and n_e is the electron density. $\pi(n)$ is defined by

$$\begin{gathered}\pi(n) = -K_x^2 n^2\left[A \ln n - B(n-1) + \beta\frac{n^2-1}{4} + \frac{1}{n} - 1\right],\\ A = 1 + a + \beta/2a\,, \quad a = M_A^2/K_Z^2\,, \quad B = A + \beta/2 - 1\,,\\ M = V/V_A\,, \quad \beta = 2C_s/V_s\,,\end{gathered} \tag{3}$$

where C_s is the speed of sound, V_s is the Alfvén velocity, M_A is the Mach number, and T is the electron temperature.

In the model considered by Yu and Shukla,[1] the wave is localized, i.e., the change in the density n caused by the wave vanishes as $\eta \to \pm\infty$, i.e.,

$$n = 1\,, \quad \frac{\partial n}{\partial\eta} = 0 \quad \text{at} \quad \eta = \pm\infty\,.$$

Extreme changes in the density n (from the unperturbed value of 1) occur at some value of η, say η_0, where

$$n(\eta_0) = N\,, \quad \left.\frac{\partial n}{\partial\eta}\right|_{\eta=\eta_0} = 0\,. \tag{4}$$

If the wave constitutes a density maximum, then $N > 1$ and, if the wave constitutes a density minimum, then $N < 1$.

From (1)–(4)

$$\pi(1) = u(1) = 0\,, \tag{5}$$

and

$$\pi(N) = u(N) = 0\,. \tag{6}$$

Also,

$$\pi(n) \leqslant 0 \tag{7}$$

or equivalently $u(n) \geqslant 0$ for $1 \leqslant n \leqslant N$ when $N > 1$ and for $N \leqslant n \leqslant 1$ when $N < 1$.

However, Yu and Shukla[1] have checked the validity of (7) only in the neighborhood of $n = 1$ and $n = N$ and not in the entire domain between $n = 1$ and $n = N$. Moreover, although Eq. (5) has been noted by Yu and Shukla,[1] they have not ascertained under what conditions it will have unique solutions for N. Thus, it appears that a more rigorous study will be worthwhile.

We are studying solutions of (1), subject to (2) and (3) and satisfying (5)–(7). To this end, we obtain from (2):

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